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TREATMENT OF MULTIAXIAL CREEP-FATIGUE BY STRAINRANGE PARTITIONING

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ABSTRACT

Strainrange Partitioning is a recently developed method for treating creep-fatigue interaction at elevated temperature. Most of the work to date has been on uniaxially loaded specimens, whereas practical applications often involve load multiaxiality. This paper will show how the method can be extended to treat multiaxiality through a set of rules for combining the strain components in the three principal directions. Closed hysteresis loops, as well as plastic and creep strain ratcheting are included. An application to hold-time tests in torsion will be used to illustrate the approach.

INTRODUCTION

Strainrange Partitioning is a method for treating elevated temperature, creep-fatigue interactions commonly encountered in high-performance components such as nuclear reactor elements, gas turbine components, rocket nozzle liners, and other equipment subjected to reversed strain cycling in the creep temperature range. Basically, the procedure involves the partitioning of the entire reversed inelastic strainrange into four generic components identified with the basic type of strain involved (i. e., creep or plasticity), and the manner in which the components of strain in the tensile half of the cycle are reversed by the compressive half (i. e., creep, creep reversed by plasticity, plasticity reversed by creep, and plasticity reversed by plasticity). The concept is that the deformation mechanisms involved in reversal types may differ from each other, resulting in possible differences in life for each of the component types even if the magnitude of the strainrange is the same. A number of reports (1-7) have been published describing the basis of the method, the characterization of a number of materials according to its framework, and its application to a number of practical problems.

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This report addresses the question of how to deal with multiaxial stress aspects of high-temperature, low-cycle fatigue using the method of Strainrange Partitioning. The report is divided into three major sections; A general discussion of the rationale used in formulating the general set of rules for treating multiaxiality; a concise listing of these rules; and an illustrative example problem involving the prediction of torsional creep-fatigue lives on the basis of axial creep-fatigue information in conjunction with the set of rules.

Most of the experimental work conducted to date in connection with Strainrange Partitioning has been on specimens subjected to uniaxial stress. Unfortunately, practical design situations for which life prediction methods are intended to be used, often involve stresses in more than one direction. Since in many cases the regions of interest are at surfaces which are not directly loaded, the stress state is often biaxial; however, triaxiality can be present in special cases. Thus, for the eventual usage of any method, some procedure must be devised to extend it to include stress multiaxiality. Ideally, also, it is desirable for the extension to be analytical, utilizing a functional procedure to interpret the multiaxiality in terms of experimental information obtained under uniaxial stress. However, the possibility of requiring specialized multiaxial tests for generating baseline data for more generalized multiaxial use should not be excluded.

Stress multiaxiality complicates the situation from the standpoint of both the physical fatigue mechanisms involved and the analysis required to account for them. It is commonly observed, for example, that under uniaxial loading the crack starts in a direction of maximum shear stress at 45° to the applied loading direction, and, after small growth, propagates in a direction perpendicular to the maximum applied normal stress. Thus, the ratio of the shear stress to normal stress is 1 in the plane wherein the crack starts, but is 0 in the plane wherein the crack propagates. Under multiaxial loading, however, it is possible to obtain any desired value of shear-to-normal stress both in the crack initiation plane and in the crack propagation plane. Since this ratio can alter both the number of cycles to initiate the crack, and the rate at which the crack grows, it is important to recognize that uniaxial tests may not contain the necessary information to permit extension to multiaxial predictions.

Anisotropy is another facet of material behavior that is exaggerated under multiaxial loading conditions. If the material has different properties along different planes or in different directions, a multiaxial stress system can single out weak planes upon which the loadings may be more severe relative to their strength than are the maximum loadings relative to the strength of the planes that must resist them. This is especially the case when the multiaxiality results from two or more loading systems which have different principal stress directions, a condition that is further aggravated by the possibility of non-proportionality of the loading systems. Here the resultant principal directions, planes of maximum shear, octahedral shear planes, and other directions or parameters entering into the computations of flow, fatigue crack growth, and fracture are continuously changing with time, thus proliferating the complexities of analysis.

It is not surprising, therefore, that many approaches have been proposed for treating multiaxial fatigue failure. In a recent paper Brown and Miller (8) cite 19 different criteria that have been investigated in the past, and conclude that all are deficient in one respect or another. They add their own theory, based on the concept that fatigue life depends on both the maximum shear strain and on the tensile strain normal

to the plane where the maximum shear strain occurs. In accordance with their theory, each uniaxial fatigue test result provides only one datum point on one curve in a family of curves that are required to completely define the multiaxial fatigue characteristics. Additional biaxial fatigue tests are required to complete the construction of the characterization curves. As presented in Ref. (8), the new method has not yet been applied to consider effects of anisotropy, mean strain, non-proportional loading and high temperature; however, the authors point to the need for further work in these areas, and indicate that tests now in progress are directed toward a start in the direction of high temperature.

Indeed, their approach could well provide the basis for future intensification of research into the many important fundamental aspects of multiaxiality under fatigue conditions. In the interim, however, there is an urgent need for a practical approach with which to treat this critical problem within the currently available technology base. The purpose of this paper is to take a first step in this direction and show how the method of Strainrange Partitioning might be used to help satisfy this established need. To avoid the uncertainties of the initiation/propagation dilemma, we shall direct our attention primarily to the initiation of an "engineering size" crack (a crack of 3-4 grain dimensions deep, or of surface dimension about 0.010 in., whichever is the smaller). We shall also consider only proportional loading, that is, situations in which the principal stresses remain in constant proportion to each other. It is hoped that the procedures described will later be extendable into the non-proportional loading range, and for other crack size ranges.

From the foregoing discussion, it is clear that multiaxiality of stress can add a multiplicity of complexities in addition to a multiplicity of stress axes about which stresses must be analyzed. Since the problems of multiaxiality have not as yet been fully resolved even in the sub-creep temperature range it is a further challenge to Strainrange Partitioning to handle the two added factors of bi-modal straining (creep and plastic) and the necessary distinction between tensile and compressive inelastic deformation. In the following we shall evolve a simple practical procedure that presents itself as having engineering viability. Obviously it will need further development, just as do procedures using other frameworks for treating creep-fatigue interaction. But from the discussion it will be clear that the Strainrange Partitioning framework is compatible with the treatment of stress multiaxiality. We shall first indicate the procedure, and then examine it in the light of several limiting cases and in connection with some recent experimental results to which the method can be applied.

BASIC CONCEPTS INVOLVED IN THE TREATMENT OF MULTIAXIALITY

BY STRAINRANGE PARTITIONING

It will be assumed that the stress and strain components in each of the principal directions can be determined at any point in the cycle through conventional mechanics analysis, computerized finite element analysis, or direct experimental observation. In other words, it will be assumed that the hysteresis loops for all three principal directions can be evaluated. Equations appropriate for this purpose are based on equilibrium, compatibility, plasticity theory, and constitutive material behavior (Ref. 9, p. 86). Constants in the constitutive equations are generally determined from uniaxial

or simple biaxial stress state tests. Therefore, the computations leading to a knowledge of the principal stresses and strains already contain within them the rheological information for separating the strainranges into their creep and plasticity components as required for Strainrange Partitioning, since the constitutive equations used to make the calculations must necessarily involve relations between stress and elastic strain, plastic strain and creep strain. Basically, therefore, if the constitutive equations are consistent with the equations used to partition the strains, it should be possible to retrieve the appropriate strain components directly from the computer memory. However, in order to present a general procedure not related to the specifics of a given calculation procedure, we shall assume here that only the stress and total (unpartitioned) strain components in each of the three principal directions are known at any instant of time within the cycle.

1. The equivalent stress and strain parameters. The first question to be resolved is how to characterize by a single parameter the net effect of the three components of stress and strain. Since the initial purpose of such characterization is to permit rheological calculations (plasticity and creep), we shall specify the Mises-Hencky equivalent stress and strain be used for this purpose (although further study may reveal an improved procedure based on other formulas for combining stress and strain effects). Thus, at each instant of time within the cycle we form an equivalent stress σ_e, and equivalent strain ϵ_e, according to the equations

$$\sigma_e = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \tag{1}$$

$$\epsilon_e = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2} \tag{2}$$

where σ_1, σ_2, σ_3 are the principal stresses, and ϵ_1, ϵ_2, ϵ_3 are the principal inelastic strains. In this discussion we shall assume that the inelastic strains are sufficiently large so as to make the elastic strains negligible from an engineering viewpoint. However, the basic procedure to be described lends itself also to the treatment of small strains. [Ref. 9 (pp. 91, 92) shows how the treatment can be altered by defining a new quantity $\epsilon_{e, t}$ analogous to ϵ_e above, but wherein the strains $\epsilon_{1, t'}$, $\epsilon_{2, t'}$, $\epsilon_{3, t}$ are the total strains, rather than the plastic strains.] The first step in the procedure is, therefore, to evaluate at each point in the cycle the equivalent stress and equivalent strain from a knowledge of the individual components of stress and strain already available from the hysteresis loops.

Whereas methods for treating multiaxiality on the basis of the Mises-Hencky relations for the sub-creep range make use of only the peak values of the equivalent stress and strain, the treatment in the creep range by Strainrange Partitioning requires the entire hysteresis loop to be known so "tensile" and "compressive" creep and plasticity can individually be identified for partitioning purposes. From the values of equivalent stress and equivalent strain at each point in the cycle, it then becomes possible to construct an equivalent hysteresis loop wherein at each instant of time the stress is the equivalent stress and the strain is the equivalent strain. However, before this can be done an algebraic sign must be assigned to both stress and strain.

The question of algebraic sign of equivalent stress and strain is important in any multiaxial analysis. As expressed in Eqs. (1) and (2), they are determined in magnitude, not sign. Consider, for example, the case of uniaxial stress loading; strains are, of course, triaxial. When components of stress and strain are substituted into Eqs. (1) and (2), the resultant equivalent stress is the uniaxial stress and the resultant equivalent strain is the uniaxial strain. However, as the real uniaxial stress goes from tension into compression during its variation in a fatigue test, the expression for equivalent stress does not know to change its sign unless somehow instructed to do so. A sine wave variation of the real uniaxial stress reflects itself into a double half-sine wave, all above the horizontal stress axis. Some instruction is therefore necessary to place the alternate half-sine waves below the axis in order to end up with the proper stress range. In Strainrange Partitioning this is especially imporant since the signs of the stress and strain enter in a significant manner in determining the life relationships. Thus, it is important to develop a convention for treating the signs of the equivalent behavior in the treatment of multiaxiality, at least to the extent that when the multiaxial treatment is applied to a case of uniaxial loading the results will degenerate properly to the known behavior for this simple case. The manner in which this is to be done will require detailed attention. However, a simple approach will now be presented, subject to later review.

2. A rule of sign for the dominant principal direction. A rule that immediately suggests itself is to give to the equivalent stress and strain the signs of corresponding stress and strain at the instant under consideration for the most important hysteresis loop of the three. When one of the components clearly dominates the loading, the decision can be unambiguous. Treatment of cases where some ambiguity exists will be discussed later; for the present we shall assume that the dominant direction is chosen on the basis of the direction having the largest computed stress range, $\Delta\sigma$. This selection should ensure that the strain in the dominant direction is driven primarily by the stress in that direction and that the strain is not an induced Poisson strain caused by strains in a transverse direction. Thus, at each instant of time the equivalent stress and strain will be computed from all the components of stress and strain according to Eqs. (1) and (2), but the algebraic signs of the stress and strain will be the same as those of the dominant component. Note, for proportional loading, that the dominant direction remains fixed over the cycle, once it is established according to the specified criterion.

Once an equivalent hysteresis loop has been established, the creep in the tensile and compressive halves of the equivalent hysteresis loop can be analytically determined as discussed earlier. The plastic flow components can then be determined by subtraction from the inelastic strain. Partitioning of the inelastic strainranges follows readily (1). If the partitioning is to be performed experimentally (4) a uniaxially loaded specimen could be programmed to traverse the same history of temperature and axial strain as required by the equivalent hysteresis loop. The computation of life from the partitioned life relations is fully discussed in previous reports, for example, Ref. (2).

3. Consideration of secondary directions. It is thus clear that once the effective hysteresis loop has been established, the mechanics of treating triaxiality is no more difficult than treating uniaxial loading. The method of establishing the magnitude of effective stress and strain at each point in the cycle is clearcut: in all cases it depends only on the three components of each, which presumably are known. However,

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attaching the proper sign to stress and strain requires special consideration. As already noted the first approach is to choose the dominant stress direction, and to gear the signs of stress and strain of the equivalent loop to those that occur in the loop of the dominant component But sometimes there is some ambiguity as to which is the dominant component, and furthermore, it cannot be automatically assumed that the other directions should not be carefully considered even if their stresses are lower. This factor will now briefly be considered both to point out the problem, to suggest a tentative criterion, and to indicate some experimental studies that may help resolve the issue.

Consider first the case of a biaxial stress. Often biaxiality is as general a situation as is encountered from a practical point of view, since regions of practical interest are near surfaces where the normal stress is zero. Figure 1 shows the two loading types of interest. In Case I the minor principal stress σ_2 has the same sign as the major principal stress σ_1. An analysis of life in which the sign of the effective stress is based on σ_2 will therefore not differ from treatment based on σ_1 since the magnitudes of equivalent stress will be the same in both cases, as will their signs. Case II, in which the principal stresses are of opposite sign, however, requires special attention.

To be specific, assume that the loading in the dominant stress direction σ_1 causes a CP type of strain. Thus in Fig. 2(a) the hysteresis loop ABCD refers to the dominant 1-direction. Along BC there is tensile creep strain, and along CDA there is compressive plasticity. If the transverse σ_2 stress is zero, there is still transverse strain ϵ_2. Creep occurs while the material is contracting diametrally, plasticity while it is expanding diametrally. Viewed diametrally, therefore, the strain assumes a PC aspect. No problem develops, of course, in interpreting the significance of this strain in the simple uniaxial case where $\sigma_2 = 0$; it is precisely the case for which CP strain is defined, hence the life must be that for the CP strain.

Consider, however, Fig. 2(b). Here we assume that the transverse stress σ_2 is finite but small relative to σ_1, say about 5-10 percent of σ_1. The hysteresis loop for the 2-direction A'B'C'D' has now opened up. Most of the plastic strain in the 2-direction is still induced as a Poisson strain arising out of loading in the axial direction. Even though the hysteresis loop A'B'C'D' has a PC appearance, it is recognized to be essentially induced transverse strain, and not sufficiently significant to make an independent analysis whereby it serves as the basis for attachment of signs to stress and strain.

At what point should we consider a hysteresis loop in one of the transverse directions to be sufficiently important to merit consideration as a signatory parameter? The question requires experimental study, but at the very least it would appear that the strain in that direction should be driven primarily by the stress in that direction, not be the induced strain of stresses in the transverse direction. For the case of biaxial stress this occurs when the stress in the transverse direction is at least half the stress in the dominant axial direction. This (for proportional loading) follows from the equation for inelastic strain (Ref. 9, p. 90).

$$\epsilon_2 = \frac{\epsilon_e}{\sigma_e}\left[\sigma_2 - \frac{1}{2}\sigma_1\right] \tag{3}$$

Consider Fig. 2(c). ABCD still represents the hysteresis loop in the axial direction. For simplicity it is shown to be the same as ABCD in (a) and (b), even though transverse stresses affect the axial deformation as well. The strain in the transverse direction ϵ_2 is composed of a component due to the application of σ_2 in the transverse direction, and a component due to the axial stress σ_1. Both components of strain are always in the same direction since it is hypothesized that σ_2 is of opposite sign to σ_1. Thus it is obvious that when $|\sigma_2| > (1/2)|\sigma_1|$, the major component of strain in the 2-direction will be due to the stress σ_2 in that direction rather than to the induced strain resulting from the axial stress σ_1. Under this condition it is conceivable that the apparent PC type of loading in the 2-direction can be meaningful, rather than being merely a reflection of the CP loading in the axial direction.

Two situations can develop wherein this question is of considerable importance. In the first it can be conceived that the strain type in the major directions is PC but that a lesser transverse load causes the transverse strain to be of the CP type. If the material is such that the CP type of strain is more damaging than PC, it should not be overlooked that the failure mode might be governed by the CP strain, even though the stress associated with this strain is not the dominant principal stress. Second is the possibility that the PC type of strain might be more damaging than the CP type (as, for example, in $2\frac{1}{2}$ Cr-1Mo steel). In this case, even though the major stress might be causing a CP strain, the lower stress associated with the PC strain might actually be the cause of the failure. Experiments are, of couse, necessary to verify these possibilities, and to quantify the point at which the signs of the secondary stresses can become governing, but until this is done, a tentative rule can be hypothesized as follows: First for the case of biaxial stress:

a. The basic parameters are the equivalent stress and equivalent strain at each instant of the cycle. These are computed according to Eqs. (1) and (2). Thus all the stress and strain components in all the three principal directions enter directly into the computation of the stresses and strains to be used in the life analysis.

b. Identify the dominant principal direction as the one having the largest range of stress $\Delta\sigma_1$. Use the signs of the stresses and strains from the hysteresis loop in this direction to provide signs to equivalent stresses and strains as computed above. Once signs have been attached to each equivalent stress and strain, construct a hysteresis loop of equivalent stress versus equivalent strain. Then determine life, either analytically or experimentally, for a material under uniaxial loading displaying a hysteresis loop identical to the one determined from the equivalent stresses and strains.

c. Now consider the other principal direction only if the stresses are of opposite sign to those of the dominant direction. If the stress range $\Delta\sigma_2$ in this direction is at least half the stress range $\Delta\sigma_1$ in the dominant direction, i.e., $\Delta\sigma_2 > 1/2\ \Delta\sigma_1$, repeat the above calculation using the signs of stresses and strains in this direction to provide the signs for the hysteresis loop of the equivalent stresses and strains. Again compute life on the basis of the new hysteresis loop.

d. Use the lower life of the two computed lives as the conservative estimate of life.

Extension of the rule for triaxiality. The above rule can now be extended by analogy to the case of triaxial stress. Again, of course, the dominant direction is identified and used to provide signs to the equivalent stresses and strains. The 2-direction is important only if its stresses are of opposite sign to σ_1, and if its stress range, $\Delta\sigma_2$ is greater than $1/2\ \Delta\sigma_1$ and $1/2\ \Delta(\sigma_1 + \sigma_3)$.

The two criteria are needed to embrace the cases wherein σ_1 and σ_3 are equal in sign and when they are opposite. When they are equal in sign the $1/2\ \Delta(\sigma_1 + \sigma_3)$ criterion derives from the generalization of Eq. (3) for triaxiality, where σ_1 is replaced by $(\sigma_1 + \sigma_3)$. When σ_1 and σ_3 are of opposite sign, it is conceivable, however, that minor transverse stress can take on more apparent significance than is warranted. For example, when σ_1 and σ_3 are equal but of opposite sign, as in pure shear, any stress in the 3-direction would seem misleadingly significant; thus we retain the $1/2\ \Delta\sigma_1$ criterion as well to insure that small insignificant stresses do not distort the apparent nature of the governing strains. If these rules are met, a calculation is made of life based on a hysteresis loop of equivalent stresses and strains which are given the signs of the stresses and strains in the 2-direction. A similar analysis is made, of course, on the basis of the 3-direction.

As before, the lowest life calculated on the basis of all those meeting the stress criteria is the estimated life for the part.

The actual calculations, although based on the concepts discussed above, do not always involve the direct construction of hysteresis loops as described. Since the basic analytical solution of the stress and strain distribution for the part being studied requires the separation of strains into its creep and plasticity components, the partitioning information is already contained in the prior analysis; it is merely a matter of combining the strains into the quantities required for the Strainrange Partitioning framework. The actual procedure will be discussed in a later section on Rules. Before doing so, however, it is desirable to discuss the basic concepts described above in relation to several commonly encountered loading patterns.

APPLICATION OF THE CONCEPTS TO SEVERAL LOADING PATTERNS

OF INTEREST

Although the concepts described require extensive experimental verification before they can be generally adopted, the features were chosen on the basis of the reasonableness of their predictions in several cases where experimental data are actually available; or where they could be anticipated with some confidence. Some of these cases will now be discussed. To make the examination explicit we shall treat principally the cases where slow loadings in one direction are reversed by rapid loadings in the opposite direction, i.e., loadings which tend to produce CP or PC strains, and determine how we would expect the material to behave under these conditions. Other types of loadings will also be briefly discussed.

Uniaxial loading. To be valid, any multiaxial procedure should degenerate properly when applied to uniaxial loading. In this case the magnitude of the equivalent stress computed by Eq. (1) becomes simply the uniaxial stress, and the equivalent strain according to Eq. (2) becomes the uniaxial strain. Thus any loading history in the axial direction will result in equivalent stress and strain in exact agreement with the axial stress and strain. Furthermore, since the only direction having stress is the axial direction, this direction only will provide the signs for the stress and strain. Thus, the hysteresis loop of the equivalent stress and strain will be identical with the conventional hysteresis loop for the axial loading for all types of loading history. The correct results will, therefore, be obtained by this method.

Multiaxial loading in the sub-creep temperature range. Before discussing results in the creep range, it is appropriate to examine how the theory would apply to low-cycle fatigue data in the sub-creep temperature range which have been published to date. Zamrik and Goto (10) have summarized the applicability of octahedral shear strain for computing life under biaxial low cycle fatigue and have concluded that its use is suitable. The octahedral shear strain and equivalent strain Eq. (2) differ only by a constant multiplier, which means that the equivalent strain criterion would be equally acceptable. In the sub-creep temperature range the inelastic strain only is required to determine life, so neither the stress nor the entire hysteresis loop is needed. Use of the procedure outlined here provides the same answer, therefore, as the octahedral shear strain approach. Of course in these cases the only type of inelastic strain generated is of the PP type; hence partitioning is not an issue. However, the important point can be made that the method as outlined degenerated properly to a correct treatment of low-cycle fatigue in the sub-creep range.

Dominant axial stress with small transverse tensile stress. Consider now Fig. 3(a) in which a dominant stress is accompanied by a small transverse stress. If the transverse stress is of the same sign as the dominant stress it need not be considered, in any case, according to the criterion cited. Even if it is of opposite sign, it will not be considered since it doesn't meet the criterion that $\Delta\sigma_2 > 1/2 \; \Delta\sigma_1$. Therefore, only the dominant stress direction will be used to provide signs for stress and strain. The hysteresis loop of equivalent stress and strain will then resemble the hysteresis loop of the dominant direction. The magnitudes of stress and strain are, however, affected to some extent by the presence of the transverse stress. Thus, life will be affected to a quantitative degree depending on the magnitude of the transverse stress and strain it produces.

Transverse stress of substantial magnitude relative to the dominant axial stress. Figure 3(b) shows the case where the transverse stress is at least half the dominant stress. If it is of the same sign it does mean that the strain in the transverse direction is primarily governed by the stress in that direction, not being mainly an induced strain of the stress in the axial direction. If the stress and strain in this direction were used to provide signs for the hysteresis loop of equivalent stress and strain, the same results would be obtained as if the dominant direction were used to provide signs. Hence, the transverse direction can be neglected in regard to a separate life calculation, although it does, of course, affect life because of the effect on equivalent stress and strain. If the transverse stress is of opposite sign to the dominant stress, the strain in the transverse direction will be governed primarily by the stress in that direction, meeting the specified criterion, and requiring a separate calculation wherein the signs of stresses and strains in the transverse direction are used to give signs to the equivalent stresses and strains.

Consider, for example, a long tension-hold in the dominant axial direction, followed by a rapid reversal to compression. The dominant direction thus perceives a CP strain. In the transverse direction the long hold is in compression (by definition here, since it is hypothesized that the transverse stress is of opposite sign to the dominant stress). Since the induced transverse strain due to axial tension is contraction, and the applied loading also produces compressive strain, the long-hold transverse compressive stress is accompanied by compressive strain, which is later rapidly reversed to tensile strain. Thus, the transverse direction is subjected to a PC type of strain.

When a hysteresis loop is constructed from the equivalent stresses and strains using the dominant axial direction for signing, the cycle will be seen as a CP cycle, while for the transverse direction it will be seen as a PC cycle. The magnitudes of the strains involved in both cases will, of course, be the same since they are calculated from the same quantities entering into the equivalent strain. The life will then depend on the relative damage of a CP or a PC strain for the particular material. If, as in some cases, the CP type of strain is more damaging, then it will govern life. But if the material is more highly damaged by a PC type of strain, then the axial stress will not really have an important influence on life beyond adding to the magnitude of effective stress and strain. In any case, the computation should lead to a conservative estimate of life, which is desirable from a safety viewpoint.

At this time we do not know whether the hypothesis as described is accurate, and whether materials will indeed behave as has been proposed. An experimental program is required to determine the validity of this approach in general, and to establish whether the $\Delta\sigma_2 > 1/2 \ \Delta\sigma_1$ criterion is appropriate or whether a multiplier other than 1/2 is better.

Equibiaxial tensile loading. A practical biaxial stress in which both principal stresses are in the same direction and of equal magnitude, as shown in Fig. 3(c), is often encountered at the center of symmetrical circular disks and other geometries. It is also important in some thermal stress problems. Here, either direction of loading can be considered dominant; the answer will be the same. If the inelastic strainrange in either of the dominant principal direction is $\Delta\epsilon_1$, the strainrange in the thickness direction is $2\Delta\epsilon_1$, and the equivalent inelastic strainrange is $2\Delta\epsilon_1$. Thus the material will have a lower life than a uniaxial specimen of same linear strain of $\Delta\epsilon_1$. It should also be noted that an equibiaxial stress may reduce the apparent ductility of the material, further reducing fatigue life. This subject is discussed briefly in the next section on Rules, but needs further study.

Note, incidentally, that while the strain is highest in the thickness direction, the governing directions are the in-plane principal directions; they provide the signs for the stresses. Thus if a CP type of loading is applied, the effective hysteresis loop will have a CP appearance even though the strain in the thickness direction has a PC character and is the largest. For a thickness stress to be meaningful it would have to be of opposite sign (180° out of phase) to the in-plane stresses, and have a range at least equal to the range of in-plane stresses, i.e., $1/2 \ \Delta(\sigma_1 + \sigma_2) = \Delta\sigma_1$.

Torsion. When the in-plane stresses are equal in magnitude and opposite in sign, as shown in Fig. 3(d), pure shear stresses exist in the diagonal direction. If the inelastic strainrange in the 1-direction is $\Delta\epsilon_1$, the inelastic strainrange in the 2-direction is $\Delta\epsilon_2$, and the inelastic strain in the thickness direction is 0. The equivalent inelastic strain is $(2/\sqrt{3})\Delta\epsilon_1$. Again, the life is lower than that for a uniaxially loaded specimen of strainrange $\Delta\epsilon_1$.

Consider now the case where σ_1 is held for an appreciable period in tension, and reversed rapidly in compression. This direction thus assigns a CP character to the effective hysteresis loop. Also, by definition, the σ_2 stress will be applied in compression for the extended period, and rapidly reversed in tension. Thus the transverse direction will assign a PC character to the imposed loading. Since both are equally dominant directions, the procedure involved in the method would produce two hysteresis loops having stresses and strains of equal magnitudes, but having opposite characters:

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one CP, the other PC. The predicted life will be the lower of the two. Thus, for materials exhibiting lower life for CP than PC strain, the governing hysteresis loop will be the 1-direction characterized by CP, but for materials in which the PC life is lower, the 2-direction will dominate the life relation. An example involving the torsional creep-fatigue results of Zamrik (7) is given later in this report.

Torsion plus axial loading. This type of loading, common in multiaxiality studies in order to extend the ranges of stress ratios experimentally achievable, is interesting from several viewpoints. First, it should be noted that the principal directions depend on the ratio of the amount of torsion to axial loading. In torsion the principal directions are at 45° to the axis, whereas the axial loading produces a dominant principal direction parallel to the axis of the cylinder, as shown in Fig. 3(e). Thus, before Eqs. (1) and (2) can be applied to determine equivalent stresses and strains, the true principal directions must be determined. This is not a difficult matter. If the loading is proportional - that is, if the torsional and axial stresses are at all times in constant ratio, then at least the principal stress directions do not rotate during the loading. But if the proportionality is not maintained (for example, by applying a torsional load that is not either in-phase or 180° out-of-phase with the axial loading), the principal directions continually change. Thus it must be emphasized that such cases require special attention. Because it is not possible to establish three invariant principal directions about which to make the type of analysis described, further work is needed to determine methods of treating such cases. Of course, similar difficulties are encountered in the treatment of such cases by methods other than Strainrange Partitioning. Furthermore, these problems emphasize anisotropy aspects because of rotations of principal directions; so this subject also requires a further study.

GENERAL RULES FOR FATIGUE LIFE ANALYSIS IN MULTIAXIAL LOADING

We shall now consider the implications of the foregoing discussion in relation to the actual process of performing the life analysis. It is assumed, of course, that an appropriate stress-strain analysis will precede the fatigue life calculation and that this analysis will be as sophisticated as the analyst is in a position to carry out. Because of the complex shapes of the components of interest, closed-form solutions are often replaced by finite-element analyses. In either case, however, the determination of the correct stresses and strains requires a series of constitutive equations involving, separately, the plastic flow and the creep components. Thus, it can be assumed that the individual strain components in the chosen coordinate system will be available within the solution that is, the creep and plasticity components of each strain will be known directly from the solution of the system of equations involved. It then becomes merely a matter of utilizing these already determined values of strain within the framework of Strainrange Partitioning to carry out the life analysis. The steps involved in the analysis are thus as follows:

Step 1. Principal stress-strain analysis

First, the stresses and inelastic strains in each of the three principal directions must be determined through analysis using constitutive relationships which reflect the cyclic nature of the problem (i.e., cyclic stress-strain properties), the influence of multiaxiality on flow resistance, and the distinction between time-independent (plas-

ticity) and time-dependent (creep) inelasticity. If it is possible to distinguish between transient and steady-state (secondary) creep, the transient portion should be regarded as plasticity and only the steady-state considered as the important creep component. If not, treat the entire time-dependent strain as creep.

Step 2. Creep and plastic strain separation

This step permits the partitioning of the strainranges for use in Step 5. Having performed the above analysis for the crucial loading cycles of interest, the important process of separating the creep and plastic strains in the three principal directions has effectively been accomplished. For porportional loading, the relative amounts of creep strain to plastic strain will be the same in all three directions.

Step 3. Equivalent strain

The next step is to combine the inelastic strains in the principal directions into a single quantity that represents the intensity of the straining level. For this purpose, we will use the equivalent strain criterion which is based on the Mises-Hencky relation:

$$\epsilon_e = \frac{\sqrt{2}}{3} \sqrt{(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2} \tag{2}$$

where: $\epsilon_{1,2,3}$ are the total inelastic strains in each of the three principal directions. Since the equivalent inelastic strain calculated by this criterion is proportional to the square root of the sum of the squares of the principal strain differences, it is normally considered a positive quantity. It is thus necessary to invoke criteria for providing an algebraic sign to the equivalent strain so that "tensile" and "compressive" strain fields can be properly distinguished.

Step 4. Dominant and secondary directions

The sign of the equivalent inelastic strain will be taken as the sign of the principal inelastic strain in the dominant direction, i.e., the direction having the greatest cyclic range of principal stress. Occasions may arise, however, when it will be necessary to consider both the dominant and a secondary direction for providing signs, as discussed earlier in this report.

Biaxial stresses. – Consider the sign of the strain in a secondary direction only if the stress in that direction is opposite in sign to the stress in the dominant direction and if the magnitude of the secondary stress range is greater than half the magnitude of stress range in the dominant direction, i.e., $\Delta\sigma_2 > 1/2 \ \Delta\sigma_1$.

Triaxial stresses. – Consider the sign of the strain in a secondary direction only if the stress in that direction is opposite in sign to the stress in the dominant direction and if both of the criteria $\Delta\sigma_2 > 1/2 \ \Delta\sigma_1$ and $\Delta\sigma_2 > 1/2 \ \Delta(\sigma_1 + \sigma_3)$ are satisfied:

Step 5. Partitioned strainranges and creep and plastic ratchet strains

The next step is to plot the equivalent inelastic strain history to determine maximum and minimum points from which "tensile" and "compressive" half cycles of loading can be identified. In order to facilitate the description of the rules for partitioning the strainranges and ratchet strains, it is convenient to consider two consecutive half cycles made up of two minima and one maximum points as shown in Fig. 4 and to make several self-evident observations.

Similar strains in the two halves of a cycle will tend to balance one another, e.g., plastic strain in the tensile half will tend to offset and pair with an equal amount of plastic strain, if available, in the compressive half. Mechanistically, this could be

thought of as a "partial healing" process wherein the compressive plastic strain is essentially undoing some of the effect of tensile plastic strain. Obviously, the amount of plastic strain is one direction that can be healed by plastic strain in the other direction can be no more than the <u>smaller</u> of the plastic strains in the two directions. The amount of this reversed plastic strain is the $\Delta\epsilon_{pp}$ strainrange and is thus equal to the smaller of the plastic strains in the tensile (positive) or compressive (negative) direction.

Applying the above argument to reversed creep strains leads to the definition of the $\Delta\epsilon_{cc}$ strainrange as being equal to the smaller of the creep strains in the tensile or compressive direction.

After balancing off the plastic strains and the creep strains, there may be an unbalanced component of strainrange. If there is excess plasticity in tension and excess creep in compression, the unbalanced strainrange $\Delta\epsilon_{pc}$ (or $\Delta\epsilon_{cp}$ if the types of strains are reversed in their directions) is equal to the <u>smaller</u> of these two remainders, and any strain that is left over after the three reversed strainranges (PP, CC, and PC or CP) have been accounted for is thus the ratchet strain, δ_p or δ_c. On this basis, it can be shown that the partitioning can be accomplished through the following rules: Referring to Fig. 4.

Let $A_2^p = \epsilon_{2,p} - \epsilon_{1,p}$

$\qquad$ = plastic strain for "tensile" half of cycle (minimum to maximum)

$\quad A_2^c = \epsilon_{2,c} - \epsilon_{1,c}$

$\qquad$ = creep strain for "tensile" half of cycle (minimum to maximum)

$\quad A_3^p = \epsilon_{2,p} - \epsilon_{3,p}$

$\qquad$ = plastic strain for "compressive" half of cycle (maximum to minimum)

$\quad A_3^c = \epsilon_{2,c} - \epsilon_{3,c}$

$\qquad$ = creep strain for "compressive" half of cycle (maximum to minimum)

$\quad A_L^p$ = larger of A_2^p or A_3^p

$\quad A_S^p$ = smaller of A_2^p or A_3^p

$\quad A_L^c$ = larger of A_2^c or A_3^c

$\quad A_S^c$ = smaller of A_2^c or A_3^c

$\quad B_2 = A_2^p + A_2^c$

$\qquad$ = inelastic "tensile" strain (point 1 to point 2)

$\quad B_3 = A_3^p + A_3^c$

$\qquad$ = inelastic compressive strain (point 2 to point 3)

$\quad B_S$ = smaller of B_2 or B_3 = inelastic strainrange $\Delta\epsilon_{in}$

then

$\qquad \Delta\epsilon_{pp} = A_S^p$

$\qquad \Delta\epsilon_{cc} = A_S^c$

$$\Delta\epsilon_{pc} = B_S - A_S^p - A_S^c \quad \text{if} \quad A_L^p = A_2^p \quad \text{and} \quad A_S^c = A_2^c$$

$$\Delta\epsilon_{cp} = B_S - A_S^p - A_S^c \quad \text{if} \quad A_S^p = A_2^p \quad \text{and} \quad A_L^c = A_2^c$$

$$\delta_p = A_L^p - A_S^p - B_S + A_S^p + A_S^c$$

$$= A_L^p + A_S^c - B_S; \quad \text{Tensile if} \quad A_L^p = A_2^p, \quad \text{Compressive if} \quad A_L^p = A_3^p$$

$$\delta_c = A_L^c - A_S^c - B_S + A_S^p + A_S^c$$

$$= A_L^c + A_S^p - B_S; \quad \text{Tensile if} \quad A_L^c = A_2^c, \quad \text{Compressive if} \quad A_L^c = A_3^c$$

Perform the above calculations using the dominant direction as the sign donor for equivalent strain. Repeat for each secondary direction as the sign donor which satisfies the triaxiality stress condition earlier in Step 4.

Step 6. Life relationships

Partitioned strainrange versus cyclic life relationships for use in life prediction can come from uniaxial results since the effective strain is defined such as to be equal to the axial strains in an axial test. It is suggested that the life relationships for tensile creep and plastic ratchet strains can be based on the linear exhaustion of ductility concept. An example of the partitioned strainrange versus life relationships for annealed 316 stainless steel at 1300° F is shown in Fig. 5 wherein all of the time-dependent strain has been considered to be creep (6). It may be desirable to alter these relationships to account for ductility reductions brought about by exposure, or environmental effects, and by triaxiality effects. Triaxial effects will be addressed in Step 8.

Compressive ratchet strains probably do not lead to rupture type failures and hence are not considered damaging from the standpoint of creep-fatigue, although geometric instabilities may result, leading to localized buckling type failures.

Step 7. - Interaction Damage Rule

For each set of calculations from Step 5, determine the corresponding lives from the life relationships of Step 6, and apply the Interaction Damage Rule (2) modified to include the ratchet terms. The damage per cycle due to all terms is therefore:

$$\text{Damage Cycle} = \frac{F_{pp}}{N_{pp}} + \frac{F_{cc}}{N_{cc}} + \frac{F_{cp}}{N_{cp}} + \frac{F_{pc}}{N_{pc}} + \frac{\delta_p}{D_p} + \frac{\delta_c}{D_c} = \frac{1}{N_f} \qquad (4)$$

where

F = strainrange fraction

$F_{pp} = \Delta\epsilon_{pp}/\Delta\epsilon_{in}$

$F_{cc} = \Delta\epsilon_{cc}/\Delta\epsilon_{in}$

$F_{cp} = \Delta\epsilon_{cp}/\Delta\epsilon_{in}$

$F_{pc} = \Delta\epsilon_{pc}/\Delta\epsilon_{in}$

$\Delta\epsilon_{in}$ = total inelastic strainrange = $B_S = \Delta\epsilon_{pp} + \Delta\epsilon_{cc} + \Delta\epsilon_{cp}$ (or $\Delta\epsilon_{pc}$)

δ_p = plastic ratchet strain per cycle

δ_c = creep ratchet strain per cycle

D_p = plastic ductility = $-\ln[1 - (RA)_p]$, where $(RA)_p$ = reduction of area in tensile test

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D_c = creep ductility = $-\ln\left[1 - (RA)_c\right]$, where $(RA)_c$ = reduction of area in creep rupture test

N_{pp} = PP life, read from PP relationship using inelastic strainrange, $\Delta\epsilon_{in}$

N_{cc} = CC life, read from CC relationship using inelastic strainrange, $\Delta\epsilon_{in}$

N_{cp} = CP life, read from CP relationship using inelastic strainrange, $\Delta\epsilon_{in}$

N_{pc} = PC life, read from PC relationship using inelastic strainrange, $\Delta\epsilon_{in}$

N_f = expected cyclic life if above cycle is repeated until failure

Solve for N_f using the dominant and secondary directions for sign donors for the equivalent strain. For conservatism, take the lowest life. A still more conservative estimate can be obtained by consideration of the triaxiality factor, as discussed below.

Step 8. Triaxiality factor

Before the life relationships, based on axial tests, are used in making finalized life predictions for multiaxial situations, recognition should be made of the ductility reductions that are created by the presence of a triaxial tensile stress state. The triaxiality factor, TF, introduced by Davis and Connelly (11) and further developed by Manjoine (12) is used herein to indicate the reduction in cyclic strain resistance introduced by tensile hydrostatic stress states.

$$TF = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}} \tag{5}$$

As a first approximation, we shall follow Manjoine's (12) suggestion that ductility be reduced by dividing by TF. It should be pointed out, however that in some cases TF can be less than unity; for example in pure torsion it is zero. If the ductility modification rule were applied to torsion there would be an implied infinite increase in ductility due to the multiaxiality. Hence, for conservatism we shall apply the rule only for TF > 1.

Reductions in ductility affect both the ratcheting and the fatigue components in the life relations. For the ratcheting components we simply reduce D_p and D_c by dividing their values determined in a uniaxial test by TF. For the fatigue components the problem is somewhat more complex. As a simple approximation, we shall assume that the effect can be introduced by dividing each life value N_{pp}, N_{cc}, N_{cp}, and N_{pc} by $(TF)^{1/m}$ where m is the appropriate exponent in the life relation $\Delta\epsilon = AN_f^{-m}$ for each of the strainrange components. For example, if the exponent in the N_{pp} relation is -0.6, and that in the N_{cc} relation is -0.8, N_{pp} will be divided by $(TF)^{1/0.6} = (TF)^{1.67}$ and N_{cc} will be divided by $(TF)^{1.25}$ before applying the Interaction Damage Rule to determine the damage component per cycle. This approach follows by analogy to the Universalized Life Relationship discussed by Manson in Ref. 2.

As pointed out by Manjoine (12), the ductility correction should depend on the strain rate sensitivity of the material. Thus, further work is required to refine the simplified rule proposed above.

Step 9. Non-proportional loading

Initially, we have considered proportional loading in which the stress (and inelastic strains) in the principal directions are proportional to each other at all times and hence pass through zero at the same time. Extension to non-proportional loading can be based on analogous rules, but will require further study.

ILLUSTRATIVE EXAMPLE AND COMPARISON OF PREDICTED

AND OBSERVED LIVES

To illustrate the use of the above rules, an example problem will be analyzed using the high-temperature torsional strain-cycling results of Zamrik (7) to compare experimental lives and predicted lives. In particular, test number 13 will be used as the example since it is representative of the most general case investigated by Zamrik. The material is AISI Type 304 stainless steel tested in completely reversed torsional strain at 1200° F. Since the specimens were thin-walled tubes which were instrumented for twist and torque measurements there was no need for the principal stress-strain analysis called out in Step 1. The stresses and inelastic strains were measured directly. Separation of the creep and plastic strains was accomplished by direct observation of the torsional hysteresis loop. Creep was introduced by holding at constant stress in both the clockwise and counterclockwise directions. All time-dependent inelastic strain was considered to be creep. An example schematic cycle of torsional stress, τ, and strain, γ, versus time is shown in Fig. 6 along with a sketch of the hysteresis loop. No ratchering is involved in this example. For torsional straining, the equivalent inelastic strain ϵ_e is related to the torsional shear strain γ by the relation:

$$\epsilon_e = 0.577\gamma$$

Torsional straining is rather unique in that there are two equally dominant directions (both 45° helix angles on the tubular specimen). The principal stress and strain hysteresis loops in these two directions would look similar in shape to the hysteresis loop shown in Fig. 6, although both the stress and strain axes would be scaled in accordance with the equivalent stress-strain criterion. The stress and strain axes of the two loops, however, would be reversed in sign one to the other. There would be no hysteresis loop in the third principal direction since both the stress and strain are zero normal to the cylindrical specimen surface. A secondary direction need not be considered since the criterion of Step 4 does not require it. For the case at hand (Fig. 7):

$$\epsilon_{1,p} = -0.00156 \qquad \epsilon_{1,c} = -0.00674$$

$$\epsilon_{2,p} = +0.00259 \qquad \epsilon_{2,c} = +0.00571$$

$$\epsilon_{3,p} = -0.00727 \qquad \epsilon_{3,c} = -0.00103$$

Thus,

$$A_2^p = 0.00415 \qquad\qquad A_2^c = 0.01245$$

$$A_3^p = 0.00986 \qquad\qquad A_3^c = 0.00674$$

$$A_L^p = 0.00986 \qquad\qquad A_L^c = 0.01245$$

$$A_S^p = 0.00415 \qquad\qquad A_S^c = 0.00674$$

$$B_2 = 0.0166$$

$$B_3 = 0.0166$$

$$B_S = 0.0166$$

And

$$\left. \begin{array}{l} \Delta\epsilon_{pp} = 0.00415 \\[2em] \Delta\epsilon_{cc} = 0.00674 \end{array} \right\} \text{(for both dominant directions)}$$

$$\left. \begin{array}{l} \Delta\epsilon_{cp} = 0.00571 \\[2em] \Delta\epsilon_{pc} = 0 \end{array} \right\} \text{(considering one dominant direction)}$$

$$\left. \begin{array}{l} \Delta\epsilon_{pc} = 0.00571 \\[2em] \Delta\epsilon_{cp} = 0 \end{array} \right\} \text{(considering the other dominant direction)}$$

$$\left. \begin{array}{l} \delta_p = 0 \\[2em] \delta_c = 0 \end{array} \right\} \text{(for both dominant directions)}$$

$$\left. \begin{array}{l} F_{pp} = 0.250 \\[2em] F_{cc} = 0.406 \end{array} \right\} \text{(for both dominant directions)}$$

$$\left. \begin{array}{l} F_{cp} = 0.344 \\[2em] F_{pc} = 0 \end{array} \right\} \text{(considering one dominant direction)}$$

$$\left. \begin{array}{l} F_{pc} = 0.344 \\[2em] F_{cp} = 0 \end{array} \right\} \text{(considering the other dominant direction)}$$

The N_{pp}, N_{cc}, N_{cp}, and N_{pc} lives for an equivalent inelastic strainrange of 0.0166 would normally be determined from the life relationships for this alloy (304 stainless) at 1200° F (650° C). Since reliable life relationships do not exist for AISI Type 304 stainless steel at this time, they are assumed to be approximated by the life relations obtained for AISI Type 316 stainless steel at 1300° (705° C). These life relationships have been used successfully by Saltsman and Halford (6) to predict the creep-fatigue lives of a large number of axial tests on both 304 and 316 stainless steel that have been reported in the literature. Hence, for an equivalent inelastic strainrange of 0.0166 (Fig. 5).

$$N_{pp} = 245$$

$$N_{cc} = 199$$

$$N_{cp} = 27.5$$

$$N_{pc} = 216$$

For the case at hand, TF = 0. Hence, the triaxiality factor will not be considered, so we can go directly to the prediction of cyclic life. The Interaction Damage Rule is written as:

$$\frac{0.250}{245} + \frac{0.406}{199} + \frac{0.344}{27.5} = \frac{1}{N_f} \quad \text{(for one dominant direction)}, \quad \underline{N_f = 64}$$

$$\frac{0.250}{245} + \frac{0.406}{199} + \frac{0.344}{216} = \frac{1}{N_f} \quad \text{(for other dominant direction)}, \quad \underline{N_f = 216}$$

The lower life is taken as the expected life, i.e., 64 cycles to failure. This is in exceptionally good agreement with the observed life of 60 cycles reported by Zamrik (7).

The above analysis was then applied to the remaining 13 tests of Ref. 7. Two sets of analyses were made, one using the Strainrange Partitioning life relationships for AISI Type 316 stainless steel determined at 1300° F as indicated in the above example problem, and the other using the PP life relationship for AISI Type 304 stainless steel (13) at 1200° F along with the CC, CP, and PC life relationships for the aforementioned AISI TYPE 316 stainless steel. Predictions for the first set of analyses were reasonably successful, being within a factor of three of the observed lives as indicated in Fig. 8(a). Note that the predictions were the least accurate for the PP type tests. Further examination reveals that better PP predictions could be obtained if the axial life relationship for PP had been determined using AISI Type 304 stainless steel tested at 1200° F. Weeks, et al (13) have reported results for PP testing under these conditions with a cyclic straining rate of 4×10^{-4} sec^{-1}, which is approximately equal to the cyclic rates employed by Zamrik in torsion. Unfortunately, reliable CC, CP, and PC life relationships are not also available for 304 stainless at 1200° F. The relationships for 316 stainless used earlier are the best estimates available at this time. Recomputation of the predicted lives was then made using the more appropriate PP life relationship. The results were improved as evidenced in Fig. 8(b).

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Overall agreement between predicted and observed cyclic lives is satisfactory and suggests that the proposed rules for treating multiaxiality aspects of creep-fatigue interaction by Strainrange Partitioning are appropriate. In fact, the accuracy of the predictions of the torsional creep-fatigue lives is comparable to the accuracy that has been established previously in evaluating the predictive capabilities of Strainrange Partitioning.

CONCLUDING REMARKS

The foregoing discussion has illustrated that it is possible to formulate relatively simple procedures for fatigue life analysis in the creep temperature range using the framework of Strainrange Partitioning as the basis. While many questions still require resolution, for example, how to treat non-proportional loading, this is likewise true of alternative methods. The illustrative example of the torsional specimen subjected to various hold-time patterns serves to demonstrate that good predictions can be made by this method for complex cases. It serves, in fact, to suggest certain critical tests that can be conducted to check the validity of this or other methods to provide suitable procedures for analysis. Since this example provides equal CP and PC loadings on elements at right angles to each other, the direction of crack initiation should depend critically on whether the CP or the PC strain type is more damaging. Thus, for a material like 316 stainless steel, the failure should initiate in the direction at 45° to the axis which develops the CP strainrange during strain-hold. A material such as $2\frac{1}{4}$ Cr-1Mo, for which the PC type of strainrange is more damaging, should develop the crack in the direction associated with PC loading. Thus, for the same loading pattern, the directions of crack initiation for 316 stainless steel should be at right angles to that for crack initiation in $2\frac{1}{4}$ Cr-1Mo steel. Such a critical experiment, involving both an ambiguity of direction for crack initiation, as well as the usual question regarding the number of cycles required to start the crack, could provide an excellent benchmark problem to test the capabilities of Strainrange Partitioning as well as other alternative methods of analysis.

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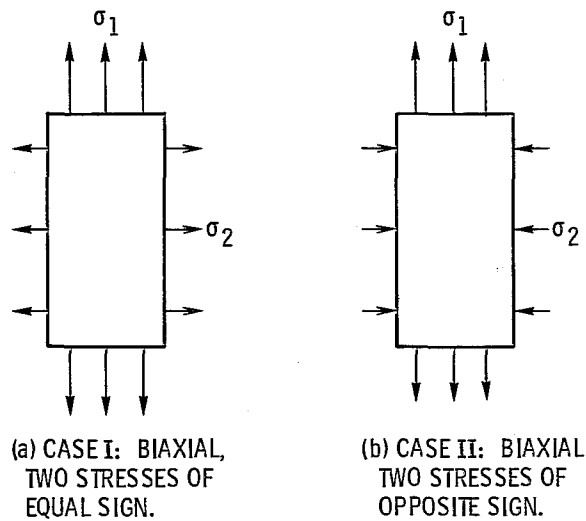
(a) CASE I: BIAXIAL,
   TWO STRESSES OF
   EQUAL SIGN.

(b) CASE II: BIAXIAL
   TWO STRESSES OF
   OPPOSITE SIGN.

Figure 1. - Two biaxial loadings to illustrate aspects of
   multiaxiality.

(a) TRANSVERSE
   STRESS: ZERO.

(b) TRANSVERSE
   STRESS:
   $|\sigma_2| \ll |\sigma_1|$.
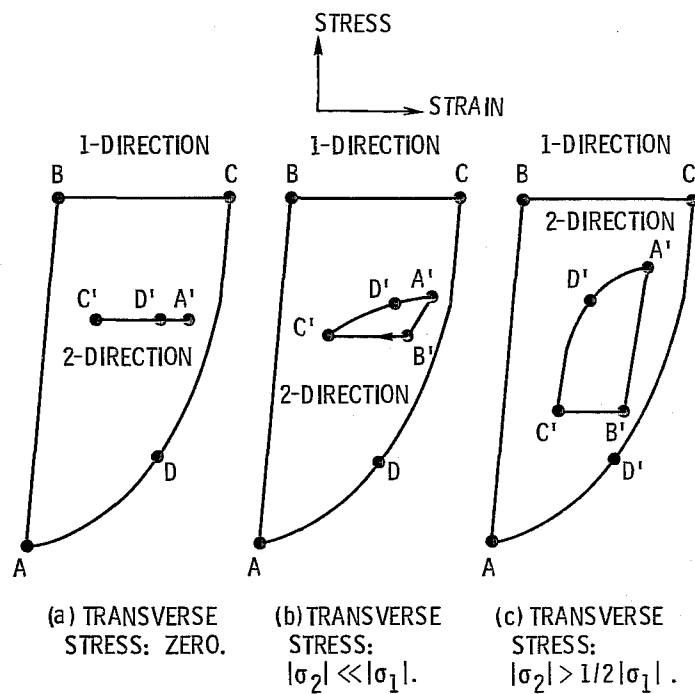
(c) TRANSVERSE
   STRESS:
   $|\sigma_2| > 1/2|\sigma_1|$.

Figure 2. - Several types of hysteresis loops for biaxial loading
   wherein the transverse stress is opposite in sign to that of
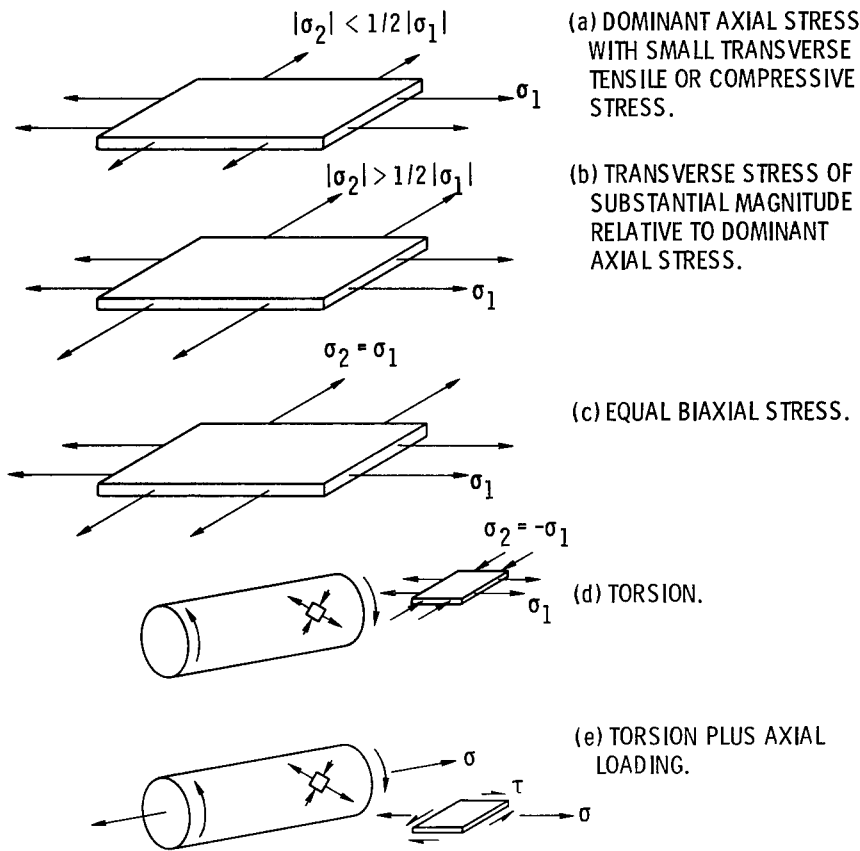   the stress in the dominant direction.

$|\sigma_2| < 1/2 |\sigma_1|$

(a) DOMINANT AXIAL STRESS WITH SMALL TRANSVERSE TENSILE OR COMPRESSIVE STRESS.

$|\sigma_2| > 1/2 |\sigma_1|$

(b) TRANSVERSE STRESS OF SUBSTANTIAL MAGNITUDE RELATIVE TO DOMINANT AXIAL STRESS.

$\sigma_2 = \sigma_1$

(c) EQUAL BIAXIAL STRESS.

$\sigma_2 = -\sigma_1$

(d) TORSION.

(e) TORSION PLUS AXIAL LOADING.

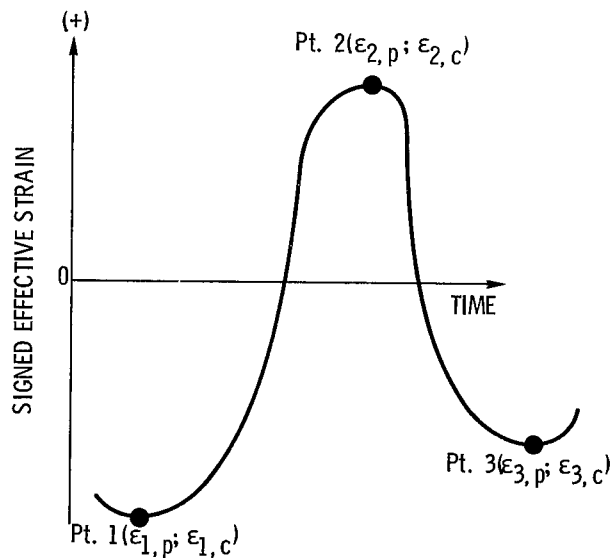Figure 3. - Several cases of biaxial loading discussed in connection with analysis by strainrange partitioning.

Figure 4. - Schematic strain cycle in terms of signed effective strain.
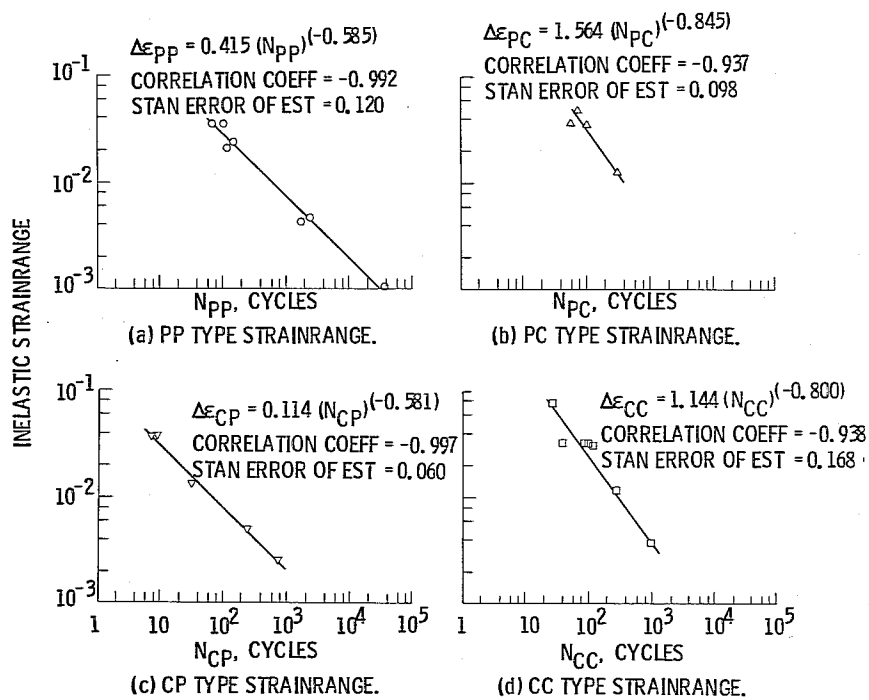
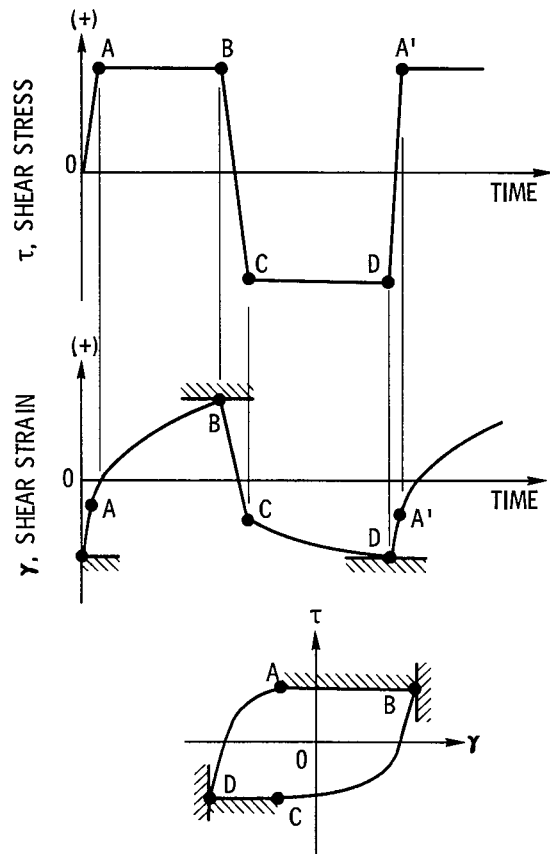Figure 5. - Partitioned strainrange - life relations for AISI Type 316 stainless steel, 1300^0 F $(705^0$ C) (6).

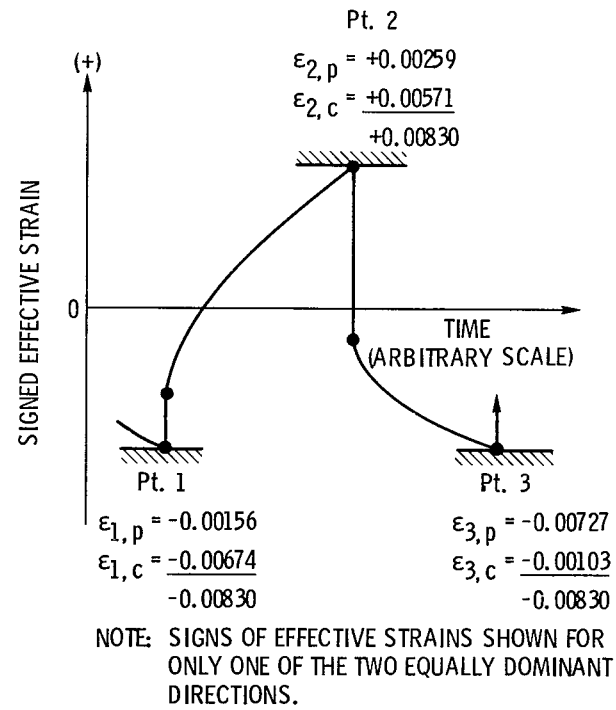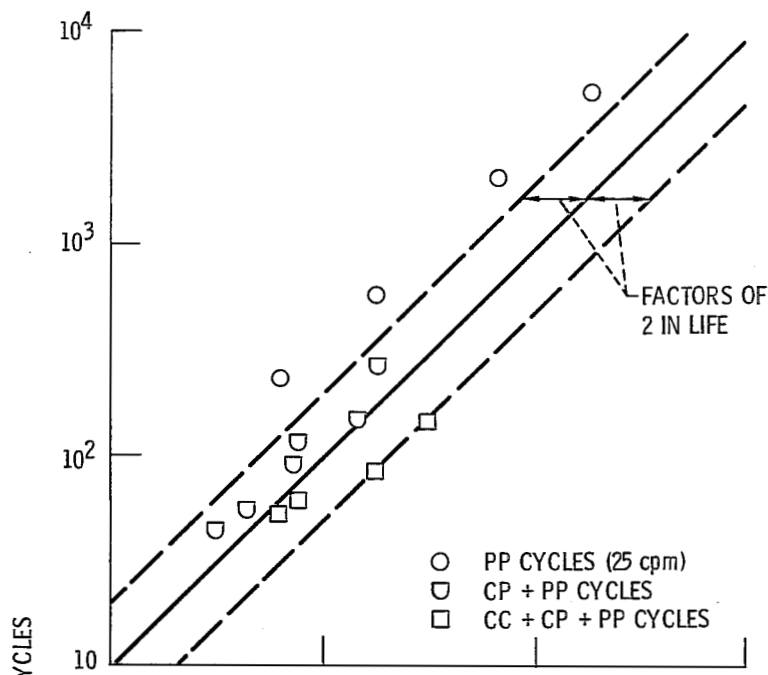Figure 6. – Schematic shear stress and shear strain conditions used by Zamrik (7).

Pt. 2

$\varepsilon_{2,p} = +0.00259$

$\varepsilon_{2,c} = \underline{+0.00571}$

$+0.00830$

Pt. 1

$\varepsilon_{1,p} = -0.00156$

$\varepsilon_{1,c} = \underline{-0.00674}$

-0.00830

Pt. 3

$\varepsilon_{3,p} = -0.00727$

$\varepsilon_{3,c} = \underline{-0.00103}$

-0.00830

NOTE: SIGNS OF EFFECTIVE STRAINS SHOWN FOR ONLY ONE OF THE TWO EQUALLY DOMINANT DIRECTIONS.

Figure 7. – Typical cycle from test 13 of Zamrik (7).

OBSERVED LIFE, CYCLES

PREDICTED LIFE, CYCLES

○ PP CYCLES (25 cpm)
▽ CP + PP CYCLES
□ CC + CP + PP CYCLES

FACTORS OF 2 IN LIFE

(a) LIFE RELATIONS BASED ON AXIAL CREEP-FATIGUE DATA FOR
AISI TYPE 316 STAINLESS STEEL AT 1300° F (705° C), (6).
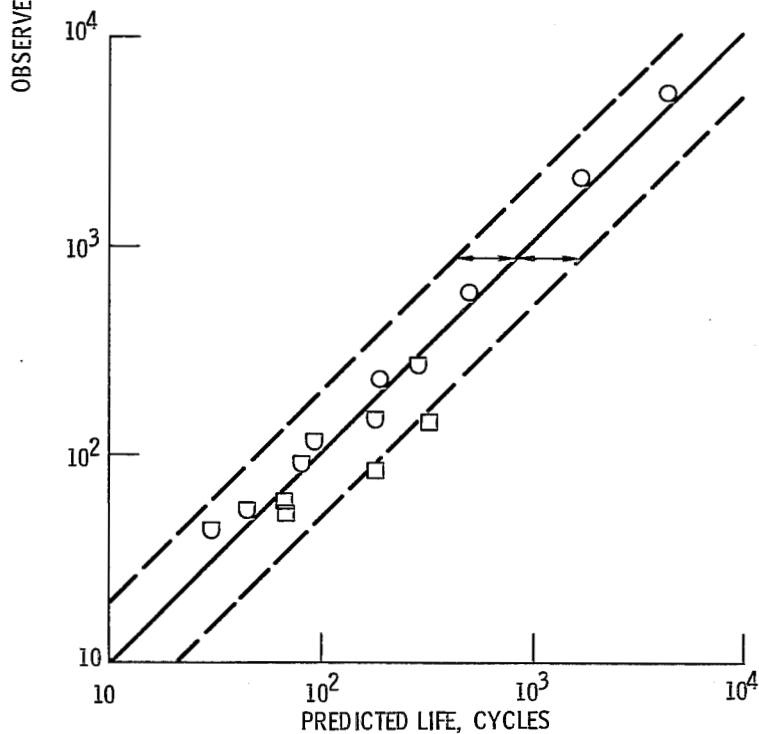
(b) LIFE RELATIONS BASED ON AXIAL CREEP-FATIGUE DATA: PP
RESULTS FROM TESTS ON AISI TYPE 304 STAINLESS STEEL AT
1200° F (650° C) (13); CC, CP, AND PC RESULTS FROM TESTS
ON AISI TYPE 316 STAINLESS STEEL AT 1300° F (705° C) (6).

Figure 8. – Applicability of strainrange partitioning multiaxiality
rules to prediction of Zamrik's (7) torsional creep-fatigue lives
for AISI type 304 stainless steel at 1200° F (650° C).